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Higher-derivative gauge interactions of Bagger-Lambert-Gustavsson theory in $N = 1$ superspace [1]

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Abstract

We study the structure of the gauge sector of the Bagger-Lambert-Gustavsson (BLG) theory in the form proposed by van Raamsdonk, adapted to 3D, $N = 1$ superspace. By using the novel Higgs mechanism proposed by Mukhi and Papageorgakis, we derive the manifestly $N = 1$ supersymmetric higher-order terms (beyond the supersymmetric Yang-Mills action) that follow from the BLG theory in its expansion with respect to the inverse gauge coupling constant squared. We find that all those terms have at least one anti-commutator of the super-YM field strength superfields as a factor, and thus are reducible to the SYM terms with the higher (spacetime) derivatives.

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1 Introduction

Unlike the notion of the *Abelian* Born-Infeld (BI) action which is well defined, the notion of a *Non-Abelian* Born-Infeld (NBI) action is rather misleading because it is dependent upon a perturbation theory, and it has to include the terms depending upon the derivatives of the fields. So it may be better to talk about particular deformation of the Yang-Mills theory by specifying the underlying theory that leads to the higher-order terms. The same observations equally apply to the supersymmetric extensions of the NBI-type actions (see eg., refs. [1, 2] for reviews). For instance, a (perturbatively defined) NBI action naturally arises as the effective action from the open superstring scattering amplitudes [3, 4, 5], whereas the Abelian BI action arises as the low-energy effective action of a single D-brane [6]. The effective action of multiple (coinciding) D-branes also contains an NBI action [7], though many attempts to explicitly construct such NBI action were not very successful (see eg., ref. [8, 9]). Another NBI action is supposed to arise as part of the effective action of multiple M-branes. [2]

Yet another ambiguity in defining an NBI action is related to the fact that (unlike the abelian case) allowing large values of the non-abelian YM field necessarily implies allowing large values of its derivatives because of the identity

$$[F_{\mu\nu}, F_{\lambda\rho}] = D_{[\mu}D_{\nu]}F_{\lambda\rho} \tag{1.1}$$

The same conclusion arises by requiring the absence of a formation of black holes in an NBI theory [14]. In other words, the presense of the higher-derivative terms in any NBI action is inevitable and model-dependent.

More recently, multiple M2-branes were investigated in the context of BLG theory [15, 16] (see also the ABJM theory [17] and ref. [18] for a recent review). Though the original BLG theory may be only applicable to two coinciding M2-branes, a variation of the Higgs mechanism arising in the BLG and ABJM theories [19] (see also ref. [20]) directly gives rise to a perturbative expansion in terms of the inverse YM coupling constant squared, g_{YM}^{-2} (or, equivalently, in terms of the inverse vacuum expectation value of the Higgs field, $\langle X \rangle^{-1}$), before the α'-corrections are taken into account. As a result, a new higher-order action arises that is truly non-abelian. That higher-order expansion is complementary to the standard α'-expansion, while they are truly independent. The α'-corrections are inherent to any D-brane action, whereas the g_{YM}^{-2} corrections are inherent to a particular background that $D2$-branes are probing.

The original BLG and ABJM actions have a high amount of supersymmetry, but we are going to concentrate on a particular (gauge) sector of those supersymmetric gauge theories by using simple (or $N = 1$) superfields in three dimensions (3D). The superfield description of the BLG and ABJM theories in 3D, $N = 1$ superspace was given in ref. [21]. [3]

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[2]As regards some specific NBI proposals, see also refs. [10, 11, 12, 13].

[3]See refs. [22, 23, 24] for the BLG/ABJM theories in terms of extended superfields in 3D.

Our paper is organized as follows. Sec. 2 is our setup devoted to a superspace description of 3D supersymmetric gauge theories. In sect. 3 we introduce our model as part of the BLG/ABJM theory. A calculation of the higher-order (and higher-derivative) gauge terms resulting from the Higgs mechanism is given is Sec. 4. Our conclusion is Sec. 5. Our notation and superspace conventions are collected in Appendices A and B.

2 Setup

An $N = 1$ supersymmetric non-Abelian gauge theory in three spacetime dimensions is defined in flat 3D superspace $z^A = (x^\mu, \theta_\alpha)$ via the Lie algebra-valued gauge- and super-covariant derivatives [4]

$$\nabla_A = D_A + i\Gamma_A \tag{2.2}$$

subject to the off-shell superfield constraints [25]

$$\{\nabla_\alpha, \nabla_\beta\} = -2i\nabla_{a\beta} \tag{2.3}$$

Equivalently, eq. (2.3) means that the vector gauge connection is not independent but can be writen down in terms of the spinor gauge connections as follows: [5]

$$\Gamma_{\alpha\beta} = \tfrac{i}{2}\left(D_{(\alpha}\Gamma_{\beta)} + i\{\Gamma_\alpha, \Gamma_\beta\}\right) \tag{2.4}$$

The supergauge connection Γ_A belongs to the adjoint representation of the gauge group. Under the supergauge transformations with the gauge Lie algebra-valued parameter K the connection transforms as

$$\delta\Gamma_A = \nabla_A K = D_A K + i[\Gamma_A, K] \tag{2.5}$$

The superspace Bianchi identities

$$[\nabla_\alpha, \{\nabla_\beta, \nabla_\gamma\}] + [\nabla_\beta, \{\nabla_\gamma, \nabla_\alpha\}] + [\nabla_\gamma, \{\nabla_\alpha, \nabla_\beta\}] = 0 \tag{2.6}$$

subject to the conventional superspace constraints (2.3) imply [25]

$$[\nabla_\alpha, \nabla_{\beta\gamma}] = -\varepsilon_{\alpha(\beta}W_{\gamma)} \tag{2.7}$$

where we have introduced the non-Abelian supercovariant superfield strtength

$$W_\alpha = \tfrac{1}{2}D^\beta D_\alpha \Gamma_\beta + \tfrac{i}{2}[\Gamma^\beta, D_\beta\Gamma_\alpha] - \tfrac{1}{6}[\Gamma^\beta, \{\Gamma_\beta, \Gamma_\alpha\}] \tag{2.8}$$

obeying the geometrical (off-shell) constraint

$$\nabla^\alpha W_\alpha = 0 \tag{2.9}$$

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[4]Our notation and conventions are collected in Appendices A and B.
[5]All (anti)symmetrizations of indices are defined without a weight factor.

The spinor superfield Γ_α can be expanded in terms of its field components as

$$\Gamma_\alpha = \chi_\alpha + \tfrac{1}{2}\theta_\alpha B + (\gamma^\mu \theta)_\alpha A_\mu + i\theta^2 \left[\lambda_\alpha - \tfrac{1}{2}(\gamma^\mu \partial_\mu \chi)_\alpha\right] \tag{2.10}$$

The supergauge transformations (2.5) can be used to impose a Wess-Zumino (WZ) gauge

$$\chi_\alpha = B = 0 \tag{2.11}$$

The remaining non-Abelian fields A_μ and λ_α can be identified as a Yang-Mills (YM) gauge field and a gaugino, resectively. The covariant superfield strength W_α takes the form

$$W_\alpha = i\lambda_\alpha + \tfrac{i}{2}(\gamma^\mu \gamma^\nu \theta)_\alpha F_{\mu\nu} - \tfrac{1}{2}\theta^2 (\gamma^\mu D_\mu \lambda)_\alpha \tag{2.12}$$

in terms of the conventional YM field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$ and the (Dirac) covariant derivative $(D_\mu \lambda)_\alpha = \partial_\mu \lambda_\alpha + i[A_\mu, \lambda_\alpha]$.

The exist *two* natural (supersymmetric and gauge-invariant) actions in 3D superspace: the super-YM action [25]

$$S_{\text{sYM}} = \frac{1}{8g_{\text{YM}}^2} \int d^3x d^2\theta \, \text{tr}\,(W^\alpha W_\alpha) = \frac{1}{g_{\text{YM}}^2} \int d^3x \, \text{tr}\,\left(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{i}{2}\lambda\gamma^\mu D_\mu \lambda\right) \tag{2.13}$$

and the super-Chern-Simons (super-CS) action [25, 26]

$$\begin{aligned}
S_{\text{sCS}} &= \frac{1}{8f_{\text{CS}}} \int d^3x d^2\theta \, \text{tr}\,\left(i\Gamma^\alpha W_\alpha + \tfrac{1}{6}\{\Gamma^a, \Gamma^\beta\}D_\beta \Gamma_\alpha + \frac{i}{12}\{\Gamma^\alpha, \Gamma^\beta\}\{\Gamma_\alpha, \Gamma_\beta\}\right) \\
&= \frac{1}{2f_{\text{CS}}} \int d^3x \, \text{tr}\,\left(-i(\lambda\lambda) + \varepsilon^{\mu\nu\rho}\left[A_\mu \partial_\nu A_\rho + \tfrac{2i}{3}A_\mu A_\nu A_\rho\right]\right)
\end{aligned} \tag{2.14}$$

It is worth mentioning that the last term of the super-CS action in 3D superspace vanishes in the WZ-gauge. As is well-known, the CS coupling constant f_{CS} gets quantized as $f_{\text{CS}} = 2\pi/k$ where $k \in \mathbf{Z}$ [27].

3   Our model

The 3D model we consider is given by an $N = 1$ supersymmetric gauge field theory with the gauge group $G \times G$ and the superfield action

$$S = S_{\text{matter}} + S_{\text{CS}}^{(1)} - S_{\text{CS}}^{(2)} \tag{3.15}$$

The matter action is given by

$$S_{\text{matter}} = \tfrac{1}{4} \int d^3x d^2\theta \, \text{tr}\,\left(\nabla^\alpha X^\dagger \nabla_\alpha X\right) \tag{3.16}$$

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with the matter superfield X in the bi-fundamental representation of the gauge group,

$$\nabla_\alpha X = D_\alpha X + i\Gamma_\alpha^{(1)} X - iX\Gamma_\alpha^{(2)} \tag{3.17}$$

where we have introduced the gauge connections $\Gamma_\alpha^{(1)}$ and $\Gamma_\alpha^{(2)}$ for each factor G in the gauge group $G \times G$. The $S_{\rm CS}^{(1)}$ and $S_{\rm CS}^{(2)}$ are the super-CS actions (2.14) for each gauge factor G.

In terms of the field components the action (3.15) represents the gauge part of the ABJM or BLG actions when $G = U(N)$ and $G = SU(2)$, respectively, in the form proposed in ref. [28]. Since our purpose is to generate a supersymmetric higher-order action for the super-YM fields, we consider only the relevant terms in what follows.

By giving an expectation value to the scalar X as

$$\langle X \rangle = \text{const.} \neq 0 \tag{3.18}$$

it is possible to spontaneously break the gauge group $G \times G$ to its diagonal subgroup $G_{\rm diag} = G$, as in ref. [19]. Then the gauge fields

$$\Gamma_\alpha = \tfrac{1}{2}\left(\Gamma_\alpha^{(1)} + \Gamma_\alpha^{(2)}\right) \tag{3.19}$$

are going to be associated with the *unbroken* gauge symmetry $G_{\rm diag}$, whereas the rest of the gauge fields

$$\Delta_\alpha = \tfrac{1}{2}\left(\Gamma_\alpha^{(1)} - \Gamma_\alpha^{(2)}\right) \tag{3.20}$$

are going to be associated with the *broken* gauge symmetry.

The Δ-dependent part of the Lagrangian in eq. (3.15) is given by

$$L(\Delta, W) = \langle X \rangle^2 \, \text{tr}(\Delta^\alpha \Delta_\alpha) + \frac{i}{2f_{\rm CS}} \text{tr}(\Delta^\alpha W_\alpha) - \frac{1}{12 f_{\rm CS}} \text{tr}(\{\Delta^\alpha, \Delta^\beta\}\nabla_\beta \Delta_\alpha) \tag{3.21}$$

where W_α is the YM superfield strength (2.12) and

$$\nabla_\beta \Delta_\alpha = D_\beta \Delta_\alpha + i\{\Gamma_\beta, \Delta_\alpha\} \tag{3.22}$$

As is clear from eq. (3.21), the supefields Δ_α are not propagating but represent the auxiliary degrees of freedom that can be eliminated via their non-dynamical equations of motion,

$$\Delta_\alpha = -\frac{i}{4\langle X \rangle^2 f_{\rm CS}} W_\alpha - \frac{1}{8\langle X \rangle^2 f_{\rm CS}} [\Delta_\beta, \nabla^\beta \Delta_\alpha] \tag{3.23}$$

By using the equations collected in Sec. 2 and the Appendices A and B, we find the bosonic (gauge) part of eq. (3.21) in the form

$$L_{\rm bos}(B, F) = 4\langle X \rangle^2 \, \text{tr}(B^\mu B_\mu) + \frac{1}{f_{\rm CS}} \varepsilon^{\mu\nu\rho} \text{tr}\left(B_\mu F_{\nu\rho} + \tfrac{2i}{3} B_\mu B_\nu B_\rho\right) \tag{3.24}$$

in terms of the vector gauge field component B_μ of Δ_α, and the YM field strength $F_{\mu\nu}$. Equation (3.24) agrees with the known results of refs. [19, 29].

4 Higher-derivative super-Yang-Mills terms

Equation (3.21) does lead to the truly Non-Abelian deformation of the YM action by the higher-order terms with the higher derivatives. Though it is impossible to solve eq. (3.23) for Δ_α in a finite explicit form, it is always possible (and easy) to get an iterative solution up to any given order in W. Substituting the iterative solution back into the action (3.21) and using the identity (2.9), we find

$$
\begin{aligned}
L(W) \;=\; & \frac{1}{2^4 \langle X\rangle^2 f_{\mathrm{CS}}^2}\,\mathrm{tr}\,(W^\alpha W_\alpha) - \frac{i}{3\cdot 2^8 \langle X\rangle^6 f_{\mathrm{CS}}^4}\,\mathrm{tr}\left(\{W^\alpha, W^\beta\}\nabla_\beta W_\alpha\right) \\
& - \frac{1}{2^{14} \langle X\rangle^{10} f_{\mathrm{CS}}^6}\,\mathrm{tr}\left(\{W^\alpha, W^\beta\}\nabla_\beta \nabla^\gamma \{W_\gamma, W_\alpha\}\right) + \mathcal{O}(\langle X\rangle^{-14} f_{\mathrm{CS}}^{-8})
\end{aligned}
\tag{4.25}
$$

The first term just represents the super-Yang-Mills Lagrangian in superspace (Sec. 2), whereas the other terms have the spinorial covariant derivatives of the YM superfield strength W. The peculiar feature of those extra terms is the presence of the anti-commutator

$$
\{W_\alpha, W_\beta\} = -\tfrac{1}{6}[\nabla^\gamma, \nabla_{\gamma(\alpha}]W_{\beta)}
\tag{4.26}
$$

that has the spacetime derivative of W. It means that all the extra terms beyond the super-Yang-Mills term in eq. (4.25) give rise to the spacetime higher-derivative contributions with respect to the YM field strength in components. The same conclusion also follows from the compoment form of the anticommutator (4.26),

$$
\{W_\alpha, W_\beta\} = -\tfrac{1}{4}(\gamma^\mu \gamma^\nu \theta)_\alpha (\gamma^\rho \gamma^\sigma \theta)_\beta [F_{\mu\nu}, F_{\rho\sigma}] + \text{ fermionic terms.}
\tag{4.27}
$$

To the end of this section we explicitly derive the bosonic (YM) contributions out of the higher-order terms in eq. (4.25). The good starting point is eq. (3.24). Varying it with respect to B^μ yields

$$
B^\mu = -\frac{1}{g}\varepsilon^{\mu\nu\rho}F_{\nu\rho} - \frac{2i}{g}\varepsilon^{\mu\nu\rho}B_\nu B_\rho
\tag{4.28}
$$

where $\frac{1}{2}\varepsilon^{\mu\nu\rho}F_{\nu\rho} = {}^*F^\mu$ and $g = 8\langle X\rangle^2 f_{\mathrm{CS}}$. Equation (4.28) is well suitable for doing iterations with respect to B^μ or expanding its solution in terms of the inverse powers of g,

$$
B^\mu = -\sum_{n=0}^{+\infty} \frac{1}{g^n} S_n^\mu
\tag{4.29}
$$

where

$$
S_0^\mu = \left(\frac{2}{g}\right) {}^*F^\mu \qquad \text{and} \qquad S_{k+1}^\mu = i\varepsilon^{\mu\nu\rho}\sum_{\substack{n+m=k \\ n,m\geq 0}} [S_{n\nu}, S_{m\rho}]
\tag{4.30}
$$

Substituting the solution (4.30) back into the action (3.24) yields the NBI action

$$L(F) = \sum_{n=2}^{+\infty} L^{(n)} \tag{4.31}$$

where

$$L^{(2)} = -\frac{1}{2^3 \langle X \rangle^2 f_{\mathrm{CS}}^2} \mathrm{tr} \left(F^{\mu\nu} F_{\mu\nu} \right) , \tag{4.32}$$

$$L^{(3)} = -\frac{i}{3 \cdot 2^6 \langle X \rangle^6 f_{\mathrm{CS}}^4} \mathrm{tr} \left([F^{\mu\nu}, F_{\nu\rho}] F^{\rho}{}_{\mu} \right) , \tag{4.33}$$

$$L^{(4)} = \frac{1}{2^{13} \langle X \rangle^{10} f_{\mathrm{CS}}^6} \mathrm{tr} \left([F_{\mu\nu}, F_{\rho\sigma}][F^{\mu\nu}, F^{\rho\sigma}] \right) , \tag{4.34}$$

$$L^{(5)} = \frac{i}{2^{15} \langle X \rangle^{14} f_{\mathrm{CS}}^8} \mathrm{tr} \left([F_{\mu\nu}, F_{\rho\sigma}][F^{\rho\lambda}, F_{\lambda}{}^{\sigma}] F^{\mu\nu} \right) , \tag{4.35}$$

$$\begin{aligned}
L^{(6)} = \ & -\frac{1}{2^{21} \langle X \rangle^{18} f_{\mathrm{CS}}^{10}} \mathrm{tr} \left([[F_{\mu\nu}, F_{\rho\sigma}], F^{\rho\sigma}][[F^{\mu\nu}, F^{\lambda\eta}], F_{\lambda\eta}] \right) \\
& -\frac{1}{3 \cdot 2^{20} \langle X \rangle^{18} f_{\mathrm{CS}}^{10}} \mathrm{tr} \left([F^{\mu\nu}, F^{\rho\sigma}][F_{\rho\sigma}, F_{\lambda\eta}][F^{\lambda\eta}, F_{\mu\nu}] \right) .
\end{aligned} \tag{4.36}$$

It is instructive to specify those equations to the case of the $G = SU(2)$ gauge group with $B_\mu = B_\mu^a \sigma^a$ and $F_{\mu\nu} = F_{\mu\nu}^a \sigma^a$, where σ^a are Pauli matrices, $a = 1, 2, 3$. Equation (3.24) then takes the form

$$L(B, F) = 8 \langle X \rangle^2 B^\mu \cdot B_\mu + \frac{2}{f_{\mathrm{CS}}} \varepsilon^{\mu\nu\rho} B_\mu \cdot F_{\nu\rho} - \frac{4}{3 f_{\mathrm{CS}}} \varepsilon^{\mu\nu\rho} (B_\mu \times B_\nu) \cdot B_\rho \tag{4.37}$$

where we have introduced the usual scalar and vector products of $SU(2)$ vectors in three dimensions,

$$A \cdot B = A^a B^a \qquad \text{and} \qquad (A \times B)^a = \varepsilon^{abc} A^b B^c \tag{4.38}$$

After substituting the iterative solution of the B-equation of motion back into the Lagrangian (4.37), we find

$$L^{(2)} = -\frac{1}{4 \langle X \rangle^2 f_{\mathrm{CS}}^2} F^{\mu\nu} \cdot F_{\mu\nu} , \tag{4.39}$$

$$L^{(3)} = \frac{1}{3 \cdot 2^4 \langle X \rangle^6 f_{\mathrm{CS}}^4} (F^{\mu\nu} \times F_{\nu\rho}) \cdot F^{\rho}{}_{\mu} = \frac{1}{3 \cdot 2^4 \langle X \rangle^6 f_{\mathrm{CS}}^4} \varepsilon^{abc} F^{\mu\nu a} F_{\nu\rho}^b F^{\rho}{}_{\mu}{}^c , \tag{4.40}$$

$$\begin{aligned}
L^{(4)} & = -\frac{1}{2^{10} \langle X \rangle^{10} f_{\mathrm{CS}}^6} (F^{\mu\nu} \times F^{\rho\sigma}) \cdot (F_{\mu\nu} \times F_{\rho\sigma}) \\
& = \frac{1}{2^{10} \langle X \rangle^{10} f_{\mathrm{CS}}^6} \left\{ (F^{\mu\nu} \cdot F^{\rho\sigma})(F_{\mu\nu} \cdot F_{\rho\sigma}) - (F^{\mu\nu} \cdot F_{\mu\nu})(F^{\rho\sigma} \cdot F_{\rho\sigma}) \right\} ,
\end{aligned} \tag{4.41}$$

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$$L^{(5)} = \frac{1}{2^{12} \langle X \rangle^{14} f_{\text{CS}}^{8}} \left[(F_{\mu\nu} \times F_{\rho\sigma}) \times (F^{\rho\lambda} \times F_{\lambda}{}^{\sigma}) \right] \cdot F^{\mu\nu}$$

$$= \frac{1}{2^{12} \langle X \rangle^{14} f_{\text{CS}}^{8}} \varepsilon^{abc} \left(F_{\mu\nu}^{a} F^{\nu\rho b} F_{\rho}{}^{\mu c} F_{\sigma\lambda}^{d} F^{\sigma\lambda d} + F^{\mu\nu a} F^{\sigma}{}_{\rho}{}^{b} F^{\rho\lambda c} F_{\mu\nu}^{d} F_{\sigma\lambda}^{d} \right) \quad,$$

(4.42)

$$L^{(6)} = -\frac{1}{2^{16} \langle X \rangle^{18} f_{\text{CS}}^{10}} \left[(F_{\mu\nu} \times F_{\rho\sigma}) \times F^{\rho\sigma} \right] \cdot \left[(F^{\mu\nu} \times F^{\lambda\eta}) \times F_{\lambda\eta} \right]$$

$$- \frac{1}{3 \cdot 2^{16} \langle X \rangle^{18} f_{\text{CS}}^{10}} \left[(F_{\mu\nu} \times F_{\rho\sigma}) \times (F^{\rho\sigma} \times F^{\lambda\eta}) \right] \cdot (F_{\lambda\eta} \times F^{\mu\nu})$$

(4.43)

$$= -\frac{1}{3 \cdot 2^{16} \langle X \rangle^{18} f_{\text{CS}}^{10}} \left\{ 4 \left(F^{\mu\nu} \cdot F_{\mu\nu} \right) \left(F^{\rho\sigma} \cdot F_{\rho\sigma} \right) \left(F^{\lambda\eta} \cdot F_{\lambda\eta} \right) \right.$$

$$+ 5 \left(F^{\mu\nu} \cdot F^{\rho\sigma} \right) \left(F_{\rho\sigma} \cdot F_{\lambda\eta} \right) \left(F^{\lambda\eta} \cdot F_{\mu\nu} \right)$$

$$\left. - 9 \left(F^{\mu\nu} \cdot F_{\mu\nu} \right) \left(F^{\rho\sigma} \cdot F^{\lambda\eta} \right) \left(F_{\rho\sigma} \cdot F_{\lambda\eta} \right) \right\} \quad.$$

5 Conclusion

It follows from eqs. (4.32) and (4.39) that the YM coupling constant is given by

$$g_{\text{YM}} = \langle X \rangle f_{\text{CS}} \tag{5.44}$$

All the terms we found beyond the (s)YM action do not have the Abelian analogue – they simply vanish in the Abelian case. Moreover, each higher-order term has at least one (anti)commutator of the (s)YM field strengths. It also implies that all those terms beyond the sYM action can be rewritten to the form with the spacetime derivatives of the (s)YM field strength. Our results in superspace agree with the earlier observations in terms of the field components [29]. It implies that the action structure of a single (Abelian) M2-brane and that of multiple (Non-Abelian) M2-branes are very different. For example, the higher-order gauge interactions considered in this paper for multiple D2-branes have no counterpart for a single D2-brane.

To avoid confusion, we would like to stress again that because of eq. (5.44) the non-abelian expansion in powers of the inverse expectation value of the Higgs field in Sec. 4 is, in fact, an expansion in terms of the inverse YM coupling constant squared (or, equivalently, in terms of the inverse Chern-Simons parameter f_{CS}), and it has nothing to do with the perturbative expansion of the DBI action of multiple D2-branes in terms of the string constant α'. In the (M-theory) brane realization of the Mukhi-Papageorgakis mechanism the relevant geometry is given by a cone whose opening angle is proportional to the Chern-Simons parameter f_{CS}, while the relevant geometry of the corresponding D2-branes is that of a thin cylinder [17, 18]. Therefore, the corrections in terms of the inverse of f_{CS} come

from moving the M2-branes away from the singularity at the top of the cone, and they describe deviations of the cylinder geometry from that of the cone. [6]

Our explicit results may also be considered as the supersymmetric generalization of the earlier results about the BLG theory [29] obtained in the bosonic case.

Acknowledgements

Appendix A: spacetime notation

Our 3D metric is $\eta_{\mu\nu} = \mathrm{diag}(+, -, -)$, where $\mu, \nu = 0, 1, 2$. We use the $SL(2, \mathbf{R})$ notation for Lorentz transformations in 3D. In particular, 3D spinors belong to the fundamental representation **2** of $SL(2, \mathbf{R})$. We use the lower case *early* Greek letters for *spinor* indices, and the lower case *middle* Greek letters for *vector* indices. We also avoid explicit writing of spinor indices whenever it does not lead to confusion.

As far as the $SL(2, \mathbf{R})$ generators are concerned, we choose

$$T^0 = \tfrac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} , \quad T^1 = \tfrac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad T^2 = \tfrac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5.45}$$

with the (anti)commutation relations

$$[T^\mu, T^\nu] = f^{\mu\nu}{}_\rho T^\rho , \quad \{T^\mu, T^\nu\} = -\frac{1}{2}\eta^{\mu\nu} \tag{5.46}$$

and the Killing form

$$\mathrm{tr}\,(T^\mu T^\nu) = -\frac{1}{2}\eta^{\mu\nu} \tag{5.47}$$

The 3D vector indices are raised and lowered with $\eta^{\mu\nu}$ and $\eta_{\mu\nu}$. It follows

$$f^{\mu\nu\rho} = \eta^{\rho\sigma} f^{\mu\nu}{}_\sigma = -\varepsilon^{\mu\nu\rho} \tag{5.48}$$

where $\varepsilon^{\mu\nu\rho}$ is 3D Levi-Civita symbol, $\varepsilon^{012} = 1$.

We define 3D (Dirac) gamma matrices $(\gamma_\mu)_\alpha{}^\beta$ by

$$\gamma_\mu = 2T_\mu , \qquad \{\gamma_\mu, \gamma_\nu\} = -2\eta_{\mu\nu} \tag{5.49}$$

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[6]The authors appreciate this comment of the referee.

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so that
$$(\gamma^\mu)_\alpha{}^\beta (\gamma_\mu)_\gamma{}^\delta = -2\delta_\alpha^\delta \delta_\gamma^\beta + \delta_\alpha^\beta \delta_\gamma^\delta \tag{5.50}$$

In addition, we have
$$\text{tr}(\gamma_\mu) = 0 \ , \qquad \text{tr}(\gamma_\mu \gamma_\nu) = -2\eta_{\mu\nu} \tag{5.51}$$

and
$$\text{tr}(\gamma^\mu \gamma^\nu \gamma^\rho) = 2\varepsilon^{\mu\nu\rho} \ , \quad \text{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 2\eta^{\mu\nu}\eta^{\rho\sigma} - 2\eta^{\mu\rho}\eta^{\sigma\nu} + 2\eta^{\mu\sigma}\eta^{\nu\rho} \tag{5.52}$$

The spinor indices are raised and lowered by the real antisymmetric symbols $\varepsilon^{\alpha\beta}$ and $\varepsilon_{\alpha\beta}$ with
$$\varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \ , \qquad \varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{5.53}$$

so that
$$\varepsilon^{\alpha\beta}\varepsilon_{\gamma\delta} = \delta_\delta^\alpha \delta_\gamma^\beta - \delta_\gamma^\alpha \delta_\delta^\beta \tag{5.54}$$

The book-keeping notation
$$(\theta\psi) = \theta^\alpha \psi_\alpha \quad \text{and} \quad (\psi\psi) = \psi^2 \tag{5.55}$$

is used for the scalar products of spinors.

A conversion between a vector V^μ and the associated bi-spinor $V^\alpha{}_\beta$ is given by
$$V^\mu = \tfrac{1}{2}(\gamma^\mu)_\alpha{}^\beta V^\alpha{}_\beta \quad \text{and} \quad V^\alpha{}_\beta = -(\gamma^\mu)_\beta{}^\alpha V_\mu \tag{5.56}$$

We also define the cousins of γ^μ as follows:
$$(|\gamma^\mu)^{\alpha\beta} = \varepsilon^{\alpha\gamma}(\gamma^\mu)_\gamma{}^\beta \ , \quad (\gamma^\mu|)_{\alpha\beta} = \varepsilon_{\beta\gamma}(\gamma^\mu)_\alpha{}^\gamma \ , \quad (\bar{\gamma}^\mu)^\alpha{}_\beta = \varepsilon^{\alpha\gamma}\varepsilon_{\beta\delta}(\gamma^\mu)_\gamma{}^\delta \tag{5.57}$$

The γ- and $\bar{\gamma}$-matrices are all traceless but not symmetric, whereas the $\gamma|$- and $|\gamma$-matrices are all symmetric but not traceless. Here are some useful identities:
$$(\theta\psi) = (\psi\theta) \ , \quad \theta_\alpha \theta_\beta = \tfrac{1}{2}\varepsilon_{\alpha\beta}\theta^2, \quad \theta^\alpha \theta^\beta = -\tfrac{1}{2}\varepsilon^{\alpha\beta}\theta^2 \tag{5.58}$$

$$\theta\gamma^\mu\psi = -\psi\gamma^\mu\theta \ , \quad (\theta\psi)(\theta\phi) = -\tfrac{1}{2}\theta^2(\psi\phi) \tag{5.59}$$

$$(\theta\xi)\lambda_\alpha = \tfrac{1}{2}(\xi\gamma^\mu\lambda)(\gamma_\mu\theta)_\alpha - \tfrac{1}{2}(\xi\lambda)\theta_\alpha \tag{5.60}$$

$$(\gamma^\mu\theta)_\alpha(\gamma^\nu\theta)_\beta = \tfrac{1}{2}(\gamma^\mu \ \gamma^\nu|)_{\alpha\beta}\theta^2 \tag{5.61}$$

$$(\gamma^\mu\theta)_\alpha(\gamma^\nu\theta)_\beta \partial_\mu \partial_\nu = \tfrac{1}{2}\varepsilon_{\alpha\beta}\theta^2 \Box \tag{5.62}$$

where $\Box = \partial^\mu \partial_\mu$.

Appendix B: superspace notation [7]

3D superspace is parametrized by (x^μ, θ_α) where θ is a Grassmann spinor. As the book-keeping notation, we use

$$\partial_\mu = \frac{\partial}{\partial x^\mu} \quad , \qquad \partial_\alpha = \frac{\overleftarrow{\partial}}{\partial \theta^\alpha} \tag{5.63}$$

It follows

$$\partial_\mu x^\nu = \delta_\mu^\nu \quad , \qquad \partial_\alpha \theta^\beta = \delta_\alpha^\beta \quad , \qquad (\partial\partial)\theta^2 = 4 \tag{5.64}$$

Grassmann integration is equivalent to Grassmann differentiation,

$$\int d\theta_\alpha = \partial^\alpha \qquad \text{and} \qquad \int d^2\theta \, \theta^2 = 4 \tag{5.65}$$

The 3D supersymmetry generators are conveniently represented in 3D superspace by

$$Q_\alpha = \partial_\alpha - i(\gamma^\mu \theta)_\alpha \partial_\mu \tag{5.66}$$

so that

$$\{Q_\alpha, Q_\beta\} = 2(\gamma^\mu|)_{\alpha\beta} P_\mu \quad , \qquad [Q_\alpha, P_\mu] = 0 \tag{5.67}$$

where $P_\mu = i\partial_\mu$ are the 3D translation generators. The 3D (flat) superspace supercovariant derivatives are defined by the relations

$$\{D_\alpha, Q_\beta\} = [D_\alpha, P_\mu] = 0 \quad \text{and} \quad \{D_\alpha, D_\beta\} = 2i(\gamma^\mu|)_{\alpha\beta}\partial_\mu \tag{5.68}$$

It is easy to verify that

$$D_\alpha = -i\partial_\alpha + (\gamma^\mu \theta)_\alpha \partial_\mu \tag{5.69}$$

obey all eqs. (5.68). Here are some useful identities:

$$D_\alpha D_\beta = i(\gamma^\mu|)_{\alpha\beta}\partial_\mu + \tfrac{1}{2}\varepsilon_{\alpha\beta}D^2 \tag{5.70}$$

$$D^\beta D_\alpha D_\beta = 0 \tag{5.71}$$

$$D^2 D_\alpha = -D_\alpha D^2 = -2i(\gamma^\mu D)_\alpha \partial_\mu \tag{5.72}$$

$$(D^2)^2 = 4\square \tag{5.73}$$

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[7]Our notaton is different from that of ref. [25].

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